Spatial Hazard Models: Limitations and Applications[1]

Brigitte Waldorf, Dept. of Geog. and Regional Development, University of Arizona, Tucson, Arizona 85721, USA, Phone: +1-520-621-1652, Fax: +1-520-621-2889, Email: bwaldorf@planet.nl

Abstract. The paper critically evaluates the literature on spatial hazard models. It augments this literature by outlining the limitations as well as the as yet unexplored potential of longitudinal modeling in a spatial setting. It argues that the literature has not fully exploited the longitudinal framework for the description of spatial point patterns and spatial relationships. In particular, the paper points out how longitudinal models can be used to describe the temporal dimension implicit in the process generating an observed point pattern. The paper also argues that –when estimating explanatory longitudinal models for spatial point patterns– the literature has ignored the conceptual problems that arise when transferring longitudinal models from the 'medium time' to the 'medium space'. Finally, it identifies the realm of applications where explanatory longitudinal models can and should be applied.

1. Introduction

Longitudinal models –also referred to as hazard models and duration models– have become an established method in regional science and related disciplines, applicable in various contexts dealing with the timing of events. While first being applied by engineers concerned about the failure of products and in biomedical research concerned about the timing of deaths following the onset of a disease, longitudinal models have subsequently also been used in the social sciences to understand the temporal dimensions of such diverse phenomena as the length of unemployment spells (e.g., James 1989, Narendranathan and Stewart 1993), duration of residence (e.g., Odland 1997, Glavac and Waldorf 1998, Davies Withers 1997), and consumer store-choice dynamics (Popkowski Leszczyc and Timmermans 1996).

At the core of these models is the non-negative random variable T, measuring the duration of a state or, equivalently, the length of time prior to a terminating event, and of prime interest is the conditional probability of an event happening at time t, given that the event has not happened up to that time. An elaborate mathematical framework has been developed to model the variations in duration T by specifying conditional failure probabilities or hazard rates of failure. The framework includes non-parametric, semi-parametric and parametric formulations of the hazard model for both continuous and discrete time scales (Lawless 1982, Kiefer 1988, Yamaguchi 1991), as well as the treatment of a variety of econometric issues, such as unobserved heterogeneities (Heckman and Singer 1985) and endogenous interactions in the form of spill-over effects and spatial externalities (Irwin and Bockstael 1998).

Just like duration, distance is also a non-negative random variable. This property has more recently been used to apply the mathematical framework of longitudinal models to spatial patterns, using distance as the endogenous random variable of interest (e.g., Odland and Ellis 1992, Esparza and Krmenec 1996). Applying the mathematical framework of hazard models to distance led to the term 'spatial duration models'.

The analogy between duration and distance (spatial duration) is straightforward and justifiable from a mathematical point of view, and issues of estimation, specification, tests and diagnostics can easily be addressed using the

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extensive methodological toolbox provided for the analysis of temporal data. It is this analogy that has prompted social scientists to herald spatial duration models as a useful complement (Odland and Ellis 1992) to traditional methods for the analysis of spatial point patterns (e.g., Diggle 1983, Boots and Getis 1988). However, the analogy between duration and distance raises questions from a conceptual perspective and leads to difficulties in the interpretation (or even misinterpretations) of spatial duration models that have not yet been addresses in this emerging field. At the same time, however, regional scientists have not fully taken advantage of the richness of the longitudinal framework, confining their analyses to the description and comparison of spatial point patterns and spatial linkages rather than utilizing the framework for the understanding of spatial processes.

This paper, therefore, critically evaluates the use and interpretation of duration models in a spatial context and explores avenues of further extension. In particular, the paper augments the literature on spatial duration models by considering three specific topics. The first issue deals with the description and categorization of spatial point patterns and asks whether spatial duration models are capable of adding to such a task. The second issue deals with the question how spatial duration models can be utilized to describe the space-time trajectory of spatial patterns and thus shift to a process-oriented perspective. The third topic identifies the conceptual problems arising in the interpretation of spatial duration models, especially the interpretation of conditional failure probabilities and hazard rates when the medium is space rather than time.

The paper is organized in four sections. Following this introduction, the second section presents a brief overview of the mathematical foundations of longitudinal models.[2] The third section reviews applications of longitudinal models in a spatial setting. Moreover, it provides critical discussions on the three issues dealing with description/categorization, pattern-generating processes, and interpretation of hazards in the medium space. The paper concludes with a summary and future research directions.

2. Mathematical Foundation of Longitudinal Models

One of the classic examples of longitudinal models refers to the length of time a person is alive, often referred to as the survival time. Implicit in this example are two states, 'alive' and 'dead', and of interest is the duration of the life span, i.e., when death occurs or, more generally, when the switch between the two states occurs.

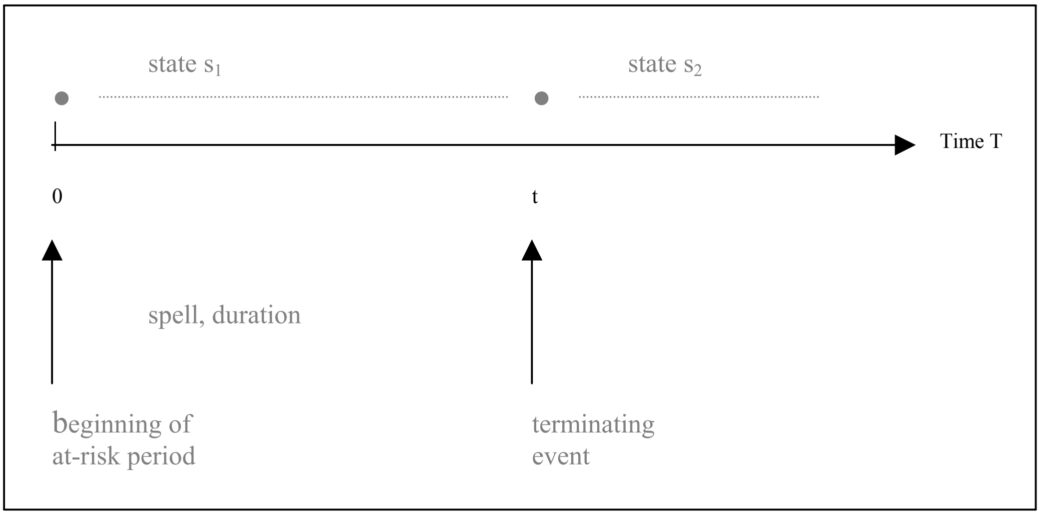
Formally, the elements of this example can be captured in the following way: let s_1 and s_2 be two states of any object or person, and let T measure the time that elapses before the switch from s_1 to s_2 occurs. As shown in Figure 1, the at-risk period begins at time T=0, and the duration of being in state s_1 or the timing of the terminating event is not constant but varies across objects/persons. When connected with a random selection process of objects/persons, T becomes a non-negative random variable whose outcomes are not known a priori.

The random variable T can be captured in a variety of well-known ways. If time is measured on a continuous scale, then the probability density function of T is a non-negative function f(x) with:

$$f : x \to f(x) \text{ such that } P(T \in (a,b)) = \int_a^b f(x)dx$$

[2] For a detailed (and very good) review, the reader is referred to Kiefer (1988).

Figure 1: Basic elements of a longitudinal design



The distribution function, F(x), specifies the probability that the survival time is less than a value x. It is defined as:

$$F(x) = P(T < x) = \int_0^x f(t)dt .$$

In the longitudinal framework, two other frequently used specifications describing the distribution of the random variable T are of importance, namely the survivor function, S(x), and the hazard rate, h(x). The survivor function describes the probability that the terminating event will not occur prior to x. Thus, it takes on the form:

$$S : x \rightarrow S(x) = P(T \geq x) = \int_x^\infty f(t)dt .$$

Note that:

$$S(x) = 1 - F(x) .$$

The hazard function, h(x), is defined as:

$$h : x \rightarrow h(x) = \lim_{\delta \rightarrow 0} \frac{P(T \in [x, x+\delta) \mid T \geq x)}{\delta} \in [0, \infty)$$

Note that:

$$h(x) = f(x)/S(x) = -\frac{d \ln S(t)}{dt}$$

The hazard function describes the exit rate or instantaneous rate of an event happening during [x,x+δ] given that it has not happened up to (and including) time x. If T is measured on a discrete scale, then f(x) is a probability function and h(x) becomes the conditional probability:

$$h(x) = \frac{f(x)}{S(x)} = \frac{P(T = x)}{P(T \geq x)} = \frac{P(T = x \cap T \geq x)}{P(T \geq x)} = P(T = x \mid T \geq x) .$$

For $dh(x)/dx > (<) \, 0$, the hazard increases (decreases). The increasing (decreasing) hazard is also referred to as positive (negative) duration dependence and implies that the conditional probability that the spell will be terminated, increases (decreases) with increasing duration of the spell.

The focus on the conditional probabilities and hazard functions distinguishes longitudinal models from the conventional regression models that, in contrast, rely on estimating the (unconditional) probability density functions, f(x). Although the probability density function and hazard function are mathematically equivalent, estimating hazard functions has two major advantages: first, information on incomplete spells (the spell lasts for at least time t but the exact duration is not known) does not need to be discarded in the estimation; second, changes in exogenous variables that occur during the spell can be accounted for by focusing on the conditional probabilities of the hazard model but not by focusing on the unconditional probability density functions of a conventional regression model.

The basic framework has been successfully applied in a variety of contexts and with a variety of estimation approaches for the hazard functions. Three types of estimation can be distinguished: non-parametric, parametric and semi-parametric approaches. The non-parametric approach is based on actuarial (life table) methods where the hazard for each discrete time interval is obtained by relating the number of failures to the duration-dependent at-risk set (those who have not yet failed at duration t), and properly accounting for censored observations.

The parametric formulations utilize a number of well-known (and well-behaved) distributions (Waldorf and Esparza 1991), such as the exponential, Weibull, log-logistic, uniform, and Gamma distribution for which the parameters are estimated via maximum likelihood techniques.[3]

The Weibull distribution with $h(x) = \lambda\beta(\lambda x)^{\beta-1}$ takes on a pivotal role as its parameters induce a flexible shape of the hazard function. For $\beta = 1$, the Weibull distribution reduces to the exponential distribution and its hazard is thus constant, i.e., the exit rates are the same no matter how long the spell has already lasted; for $\beta > (<)\ 1$, the exit rates monotonically increase (decrease) with increasing duration of the spell.

In the social sciences, the effects of exogenous variables X on the hazard are most often accommodated in a semi-parametric model such as the frequently used proportional hazard model. The proportional hazard model assumes that the hazard function is separable into two factors: a baseline hazard, $h_o(t)$, and a function $\Phi(X,\beta)$ that is independent of duration t. Typically, the function Φ is specified as an exponential function of a linear predictor such that

$$h(t \mid X) = h(t \mid X = 0)\exp\sum_{k=1}^{n}\beta_k X_k(t),$$

where $h(t \mid X=0) = h_o(t)$ is the baseline hazard and X_k are exogenous variables with associated proportional and duration-independent effect β_k on the conditional probability of terminating the spell. The baseline hazard may remain unspecified and the model is then estimated via a partial log-likelihood function[4] (Cox 1972, 1975). Alternatively, the baseline hazard may refer to a particular distribution of T (e.g., a Weibull distribution) and estimation of the parameters involves maximum likelihood procedures.

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[3] The log-likelihood function is $\ln L(\theta) = \sum_{i=1}^{n} I_i \ln f(t_i,\theta) - \sum_{i=1}^{n}(1-I_i)\ln S(t_i,\theta)$ where θ is the parameter vector, I_i is an indicator variable with $I_i=1$ if the observed duration t_i for individual i is not censored and $I_i=0$ otherwise.

[4] The partial log-likelihood function takes on the form:

$$\ln L(\beta) = \sum_{i=1}^{n}[\ln\Phi(x_i,\beta) - \ln\sum_{j=i}^{n}\Phi(x_j,\beta)].$$

Three types of covariates can be incorporated: attributes that do not change over time (e.g., race); time-dependent[5] variables that are measured for an individual but that may change during a spell (e.g., marital status); and time-varying covariates that constitute explicit functions of time, x(t).

Two frequently employed extensions of the longitudinal framework are competing-risks models (e.g., Hachen 1988, Han and Hausman 1990; Narendranathan and Stewart 1993, Thomas 1996) and multi-episode models (e.g., Blossfeld and Hamerle 1989, Popkowski Leszczyc and Timmermans 1996). The competing-risks model is appropriate if a spell can be terminated in two or more ways. For example, death may be the result of any of the many causes of death. The hazard rate of competing-risks models reduces to the sum of the single-risk hazards if the risks are assumed to be independent.[6] Multi-episode models are appropriate if more than one spell is observed for each object/individual in the sample. Most empirical studies assume restrictive conditions such as independence across spells, and thus effectively reduce the multi-episode model to a single-spell model.

3. Applications of the Longitudinal Framework in a Spatial Setting

The above section shows that longitudinal modeling has become a rich methodological tool kit that focuses on the conditional probabilities / hazards for the non-negative random variable 'duration'. Over the last decade, regional scientists have adopted the longitudinal framework in the context of spatial analysis, describing spatial hazards rather than temporal hazards. Common to all these studies is that they use distance[7] between points –a one-dimensional projection of two-dimensional space– to serve as the mathematical equivalent of duration. The emerging models are labeled spatial duration models or spatial hazard models. Since distance, just like time, can be viewed as a non-negative random variable, the entire mathematical apparatus developed for longitudinal models can also be applied to spatial duration models (Figure 2). It should be noted that this differs from the transfer typically employed in spatial econometrics. That is, whereas spatial econometrics is concerned about dealing with spatial interdependencies that arise when using spatial rather than temporal units of observation, the focus here is on using a spatial rather than temporal endogenous variable.

Although all studies using hazard models to analyze spatial patterns and processes have exclusively relied on some form of distance as the endogenous variable, two conceptually distinct types of application have emerged in the literature. The first type refers to the analysis of a spatial point patterns in a bounded area, the

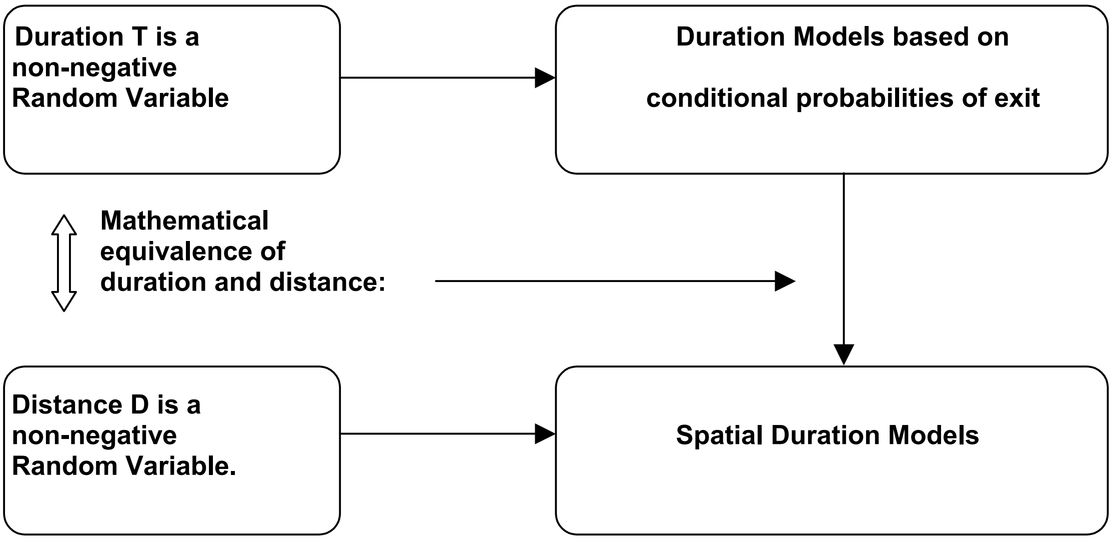
Figure 2: Transfer of Hazard Models from a Temporal to a Spatial Setting

[5] The terminology employed in the literature is inconsistent. For example Greene (1998, 2000) uses the terms 'time-dependent covariate' and 'time-varying covariate' opposite to the definitions provided in this paper.

[6] If it cannot be assumed that the competing risks are independent risks, the estimation of the joint distribution is only possible under additional restrictions (Heckman and Honoré 1989).

[7] The longitudinal framework that has been used to model distance, can also be employed for other non-negative random variables describing space, most notably area. In many applied research questions, such as those dealing with economic trade, residential search, or ecology (e.g., spread of a wild fire), the focus on area promises to provide an insightful avenue.

Figure 2: Transfer of Hazard Models from a Temporal to a Spatial Setting



second type refers to the analysis of spatial processes that manifest themselves in the form of pairs of points anywhere in space.

The study by Odland and Ellis (1992) is an example of applying the longitudinal framework to the analysis of spatial point patterns. They use a proportional hazard model with spatially varying covariates to analyze directional variations in the spacing of settlement locations in Nebraska. The variable of interest in their study is the nearest neighbor distance between settlements, and it is found that the distances increase as one moves from East to West. The Odland and Ellis study is of particular importance since it can be credited as the first attempt to utilize hazard models to study spatial patterns.

Nearest neighbor distances are also used in the Harvestore diffusion analysis by Pellegrini and Reader (1996). In addition to a proportional hazard model, they also estimate fully parameterized models where the baseline hazard is specified using various distributions, with the Weibull distribution providing the best fit. Their study contributes to the literature by demonstrating how censoring can be used to deal with edge effects.[8] Pellegrini and Grant (1999) use both nearest neighbor distances and extreme-value distances in ideological space to estimate explanatory models of policy coalitions in U.S. Congress.

A purely descriptive study of a point pattern is provided by Reader (1998). His study tackles the random labeling hypothesis and uses inter-event distances (between-point distances) rather than nearest neighbor distances. He argues that the emerging non-parametric survival functions provide a useful complement to K-function analyses of comparing spatial point patterns. It should be noted that, by using inter-event distances rather than the subset of first-order nearest neighbor distances,

[8] For a point that is closer to the edge of the study area than to any other point of the point pattern, the nearest neighbor distance is recorded as censored and as being at least as long as the distance to the edge.

his models are equivalent to multi-episode models and thus implicitly assume independence of distances in a spatial system with complex dependencies.

The second type of applications refers to pairs of points that are intricately linked, e.g., by trade or by relocation. There are two examples that fit this type, and both have the sole purpose of description, rather than explanation. First, Rogerson, Weng and Lin (1993) analyze the spatial separation between locations of parents and locations of their adult children by fitting Weibull distributions to the observed distances. Second, Esparza and Krmenec (1994, 1996) analyze the spatial extent of producer service markets by fitting Weibull distributions to the observed distances between producer services providers and their clients. They find that, for independently owned firms, the spatial hazards of distances to their clients show a negative duration dependence, whereas for non-independently owned firms (e.g., branch firms) the geography of interaction is less clustered and the hazards of spatial interaction are nearly constant (exponential distribution). As a result, the survivor function of firm-client distances for independently owned firms is steeper than that for non-independently firms. It should be noted that, since a firm may have more than one client, the data structure gives rise to a multi-episode model.[9]

In the following sections, the application of the longitudinal framework to analyze spatial relationships will be critically discussed. Specifically, three issues will be addressed: the description and categorization of spatial point patterns; the need to focus on point patterns as the outcome of a point-generating process; and finally the interpretation of conditional probabilities and hazards in the medium space.

3.1 Description and Categorization of Spatial Point Patterns

There is no doubt that, for descriptive purposes, distance data extracted from a spatial point pattern can be adequately and fruitfully analyzed in a longitudinal framework. In particular, the framework offers a convenient means of estimating the parameters for a variety of distributions for non-negative data (see section 2) and – through its ability to deal with censored data– it can efficiently handle edge effects of point patterns in a bounded area.

At the core of spatial duration models for the description of spatial point patterns is a set of points (e.g., settlements) with each point constituting an observation and the variable of interest is the distance to another point (e.g., nearest neighbor) in the point pattern.[10] To illustrate the distance dependent behavior of hazards when analyzing nearest neighbor distance, Figure 3 shows a simple example of 15 points in a 3×3.5 area, arranged in a uniform, random and clustered fashion, respectively. The empirical survivor and hazard functions of the nearest neighbor distances are shown in Figure 4. As expected, the survivor function steeply declines for the clustered pattern, reaching the median at a distance of $d_{Median} = 0.4$ already. In contrast, the survivor function of the uniform distribution is flatter and the median is

[9] Since the sample is based on randomly selected firms rather than randomly selected trade linkages, the implicit assumption of independence of multiple trade linkages measured for pne firm is of course quite strong. However, it should be noted that this dependency problem is not confined to spatial applications of hazard models. It is, for example, frequently found in (temporal) hazard models of residential mobility where more than one residential spell for a household is included in the model.

[10] Using inter-event distances rather than nearest neighbor distances, implies that each point represents n-1 observations, yielding a multi-episode model.

Figure 3: Hypothetical Point Patterns

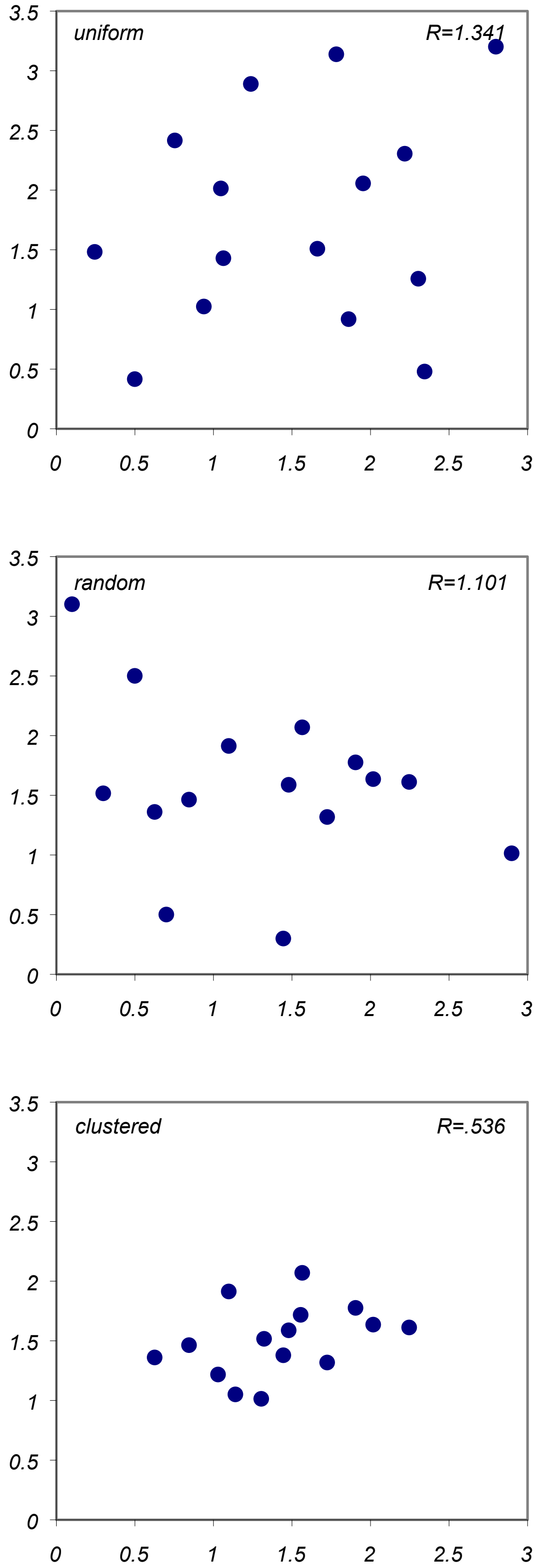
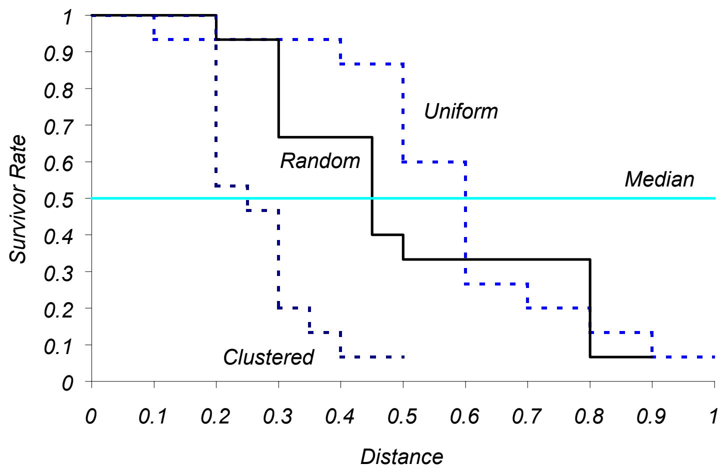
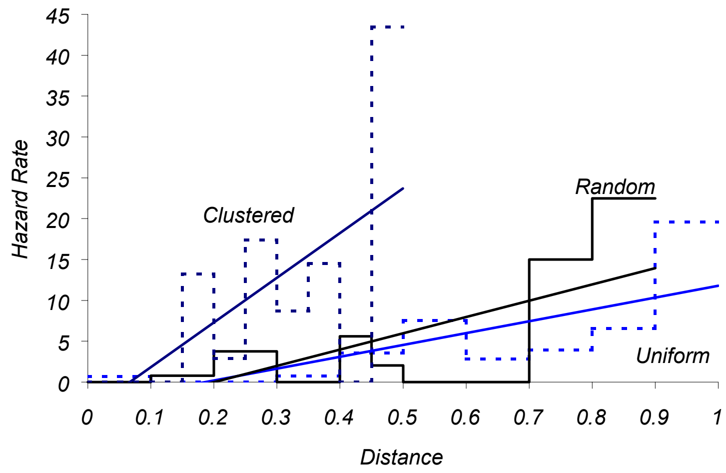


Figure 4: Non-parametric Estimated Survival and Hazard Functions for Nearest Neighbor Distances





reached at a distance of d_{Median} = 0.6. The survivor function for the random pattern takes on a middle position. Comparing the three hazard functions shows that the hazards are smallest for the uniform pattern and largest for the clustered pattern. Moreover, although the hazards show an erratic behavior,[11] the overall trends are positive for all three patterns.

This positive duration dependence suggests that fitting a Weibull distribution to the observed nearest neighbor distances will yield shape parameters $\beta > 1$ for all three patterns, and this is indeed the case (Table 1). Figure 5 shows the estimated Weibull survivor and hazard functions for the three patterns. The steepness of the estimated Weibull survivor function is most pronounced for the clustered pattern, and least pronounced for the uniform pattern, with estimated median nearest neighbor distances of d_{median} = 0.22, d_{median} = .44 and d_{median} = .54 for the clustered, random and uniform pattern, respectively. Moreover, as suggested by the scale parameters λ, the

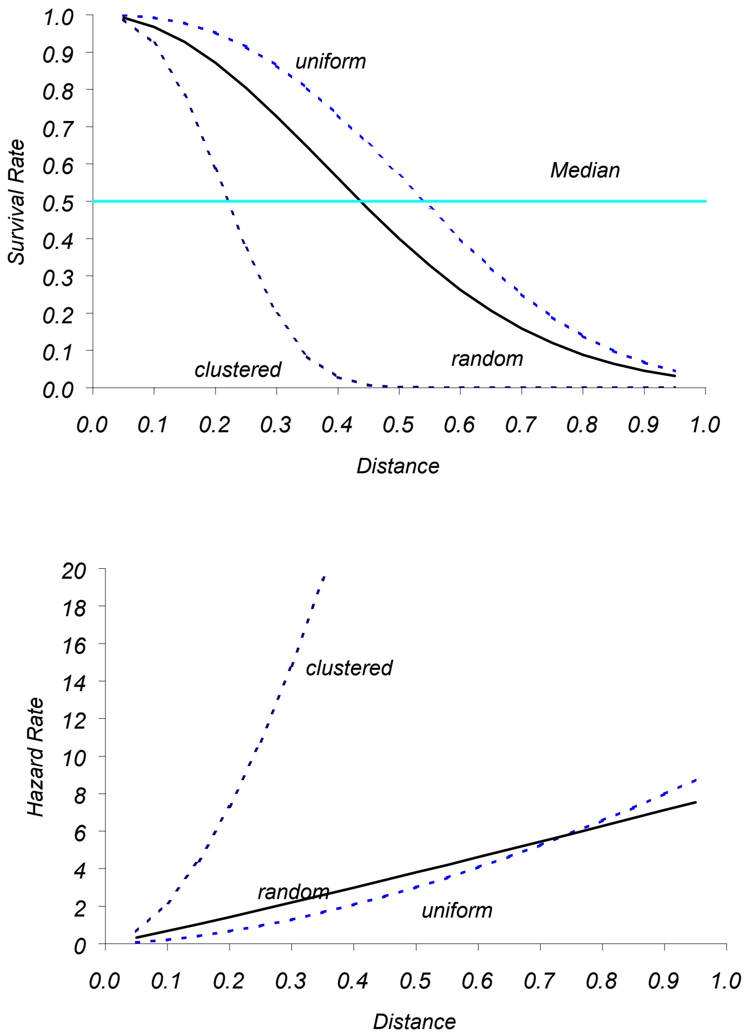
[11] Note that –since the risk set decreases with increasing distance– the number of observations used to estimate the hazard also decreases with increasing distances.

estimated (Weibull) hazards of the clustered pattern exceed the estimated (Weibull) hazards of the uniform distribution.

Table 1: Estimates of Fitted Weibull Distributions (standard errors in parentheses)

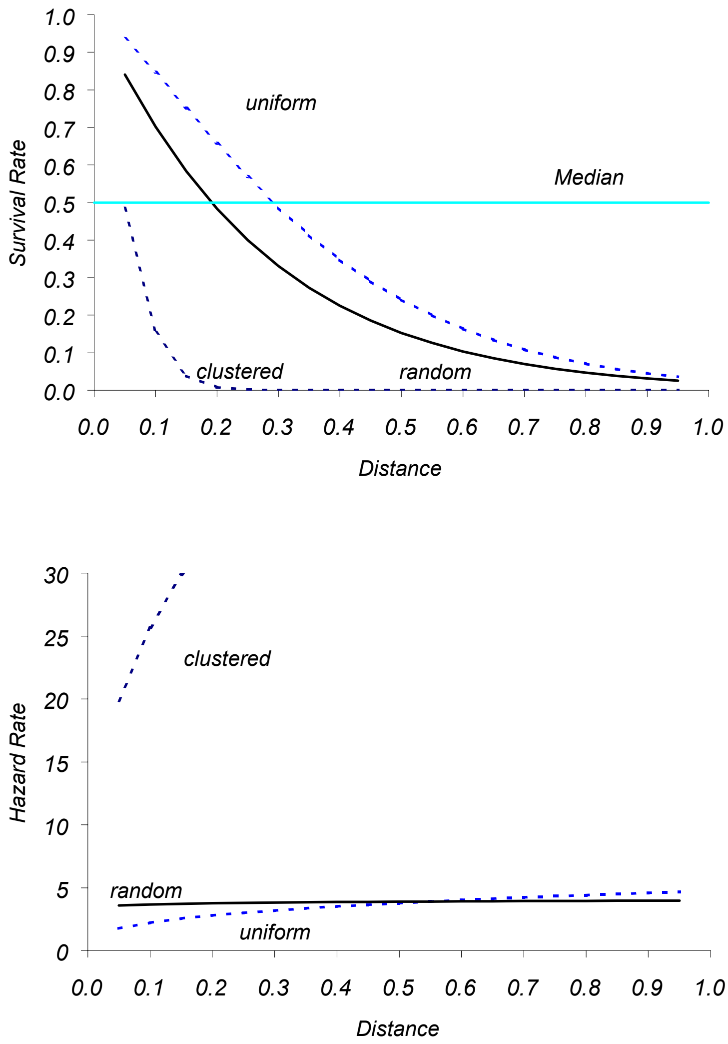
	Nearest Neighbor Distances			Squared Nearest Neighbor Distances		
	Uniform	**Random**	**Clustered**	**Uniform**	**Random**	**Clustered**
Scale: λ	1.614 (.174)	1.917 (.252)	3.973 (.571)	2.604 (.563)	3.676 (.966)	15.788 (4.536)
Shape: β	2.660 (.584)	2.070 (.781)	2.744 (.820)	1.330 (.182)	1.035 (.390)	1.372 (.410)

Figure 5: Estimated Weibull Survivor and Hazard Functions for Nearest Neighbor Distances



It is important to emphasize that positive duration dependence can occur for the nearest neighbor distances of all three types of patterns, and thus the shape of the hazard function is not suitable for a unique categorization of spatial point patterns.[12] It is, however, possible to categorize point patterns using the hazard function for the *squared* nearest neighbor distances. That is, in a random pattern, the squared nearest neighbor distances $u = d^2$, are exponentially distributed (Cliff and Ord 1981) and thus have a constant hazard:[13]

Figure 6: Estimated Weibull Survivor and Hazard Functions for Squared Nearest Neighbor Distances



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[12] In contradiction to Pellegrini and Reader (1996, p. 237) and Pellegrini and Grant (1999, p. 61), positive duration dependence cannot be interpreted as an indication of clustering and negative duration dependence does not imply decreased clustering.

[13] In the past, nearest neighbor distances rather than squared nearest neighbor distances have erroneously been associated with a constant hazard (Odland and Ellis 1992, p. 101; Pellegrini and Reader 1996, p. 224).

$$S(u) = \exp(-\lambda u\ ) \text{ and } h(u) = \lambda\ .$$

A simple test for randomness thus involves fitting a Weibull distribution with hazard $h(x) = \lambda\beta(\lambda x)^{\beta-1}$ to the squared nearest neighbor distances and testing for $\beta = 1$. As shown in Table 1, the shape parameter β of the random pattern is not significantly different from 1, and the estimated hazard function for squared nearest neighbor distances in Figure 6 shows a (nearly) constant value of $\lambda = 3.676$.

Unfortunately, such a simple test to distinguish between a clustered and a uniform pattern does not exist. However, when fitting a Weibull distribution to observed nearest neighbor distances, a negative duration dependence (i.e., a shape parameter $0 < \beta < 1$) gives rise to exponentially declining probability density function for the nearest neighbor distances and is thus a sufficient albeit not necessary condition for clustering. Furthermore, the fitted distributions can be used to assess the relative degree of clustering by comparing the estimated medians and survivor functions. Survivor functions with a small median nearest neighbor distance are indicative of a higher degree of clustering than survivor functions with a larger median.

3.2 Process-oriented Conceptualization of Spatial Point Patterns

The Odland and Ellis (1992) study suggests an East-West trend with a tendency of regular spacing of settlements. Given that the settlement process in the midwestern and prairie states of the U.S. by and large advanced from East to West, this result can also be interpreted as a time trend, with younger settlements being further apart than older settlements. As such, the results allude to the necessity of analyzing spatial point patterns as the outcomes of processes.

Spatial point patterns do not emerge instantaneously but evolve over time. This switch in perspective from patterns to processes is of particular importance in the analysis of diffusions or the spread of an epidemic. For example, high hazards at small distances, given small values of time, will be indicative of a highly contagious disease. The importance of time in the longitudinal analysis of a spatial point pattern was also mentioned, albeit not accounted for, in the diffusion study by Pellegrini and Reader (1996).

The advantage of accommodating the time dimension in a spatial duration model is that it does not require us to dissect and separately analyze the point pattern at different points in time. Formally, let j denote a point (e.g., settlement, location of an infected person) in space that is generated at time t. The spacing of j can only be evaluated relative to the already existing points at time t, and the random variable of interest becomes the nearest neighbor distance at time t, d_{jt}. Note that d_{jt} may differ from the typically used d_{jt*}, where t* denotes present time. Time T becomes an exogenous variable measured for each point and it can be entered in the linear predictor of both the partially and fully parameterized duration models:

$$h(d \mid \mathbf{z}) = h_o(d)\exp(\mathbf{z}\boldsymbol{\beta})$$

where d measures the nearest neighbor distance at the time of emergence, and the vector of exogenous variables **z** includes the time of a point's emergence.[14]

[14] Note that this model specifies distance D as the endogenous variables and time T as the exogenous variable. It is, of course, possible to reverse the assignment and specify a model in which time (e.g., in the form of duration since the beginning of the process) is the endogenous variable of interest and the spatial dimension becomes the exogenous variable. When conceptualizing both the spatial and temporal dimensions as endogenous, a proper

Figure 7: Trajectory towards Random Pattern and Nearest Neighbor Distances d_{it} and d_{it*}

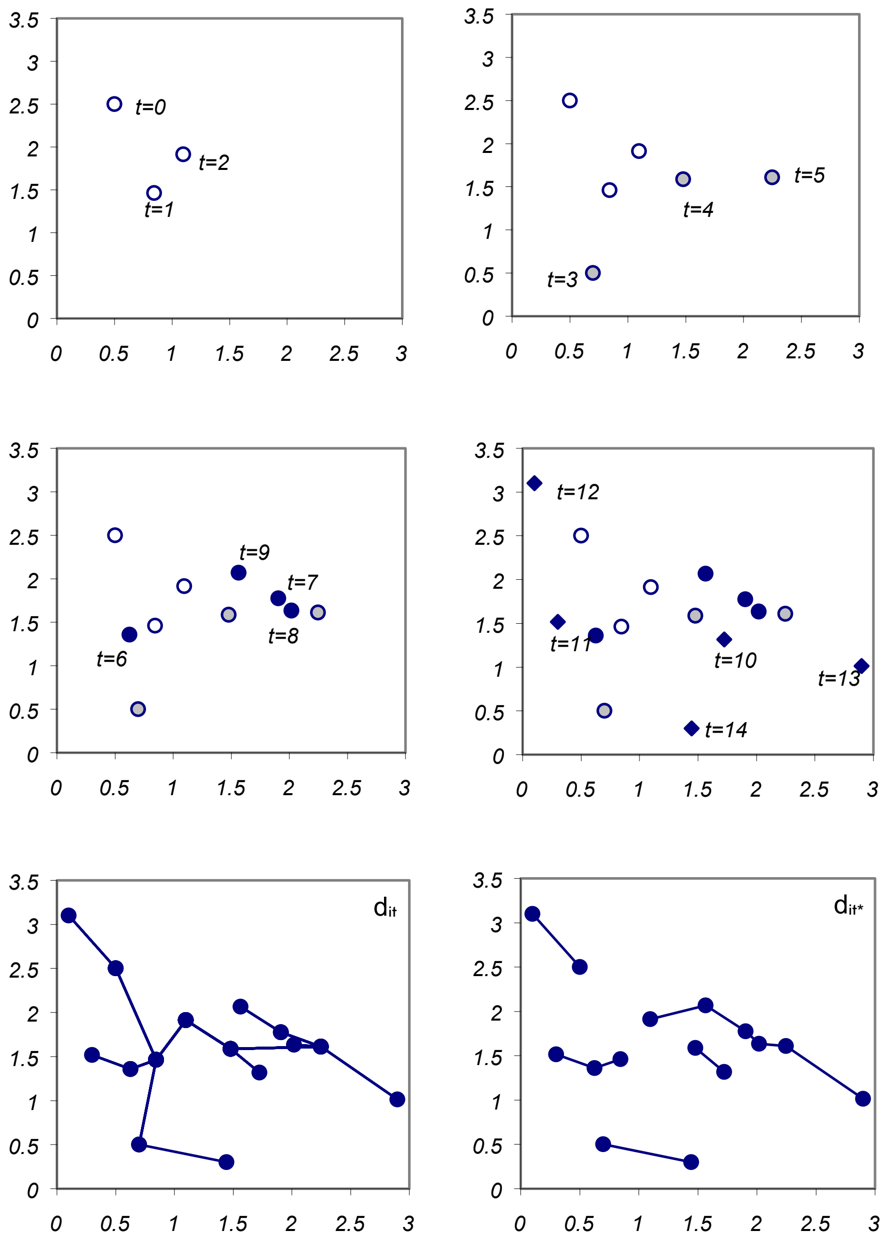

Figure 7 shows one possible trajectory towards the random pattern represented in Figure 3, the resulting time-dependent nearest neighbor distance d_{jt} for a point j at the time of its emergence, t. For comparative purposes, Figure 7 also shows the nearest neighbor distances at the end of the point generating process, d_{jt*}. Note that $d_{jt} \leq d_{jt*}$. Adding the temporal component necessitates not only a specification of the trajectory, but also of the speed of the process. The estimated hazards shown in Figure 8 assume that the process occurs at a constant speed, with one point added in

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model needs to specify the joint probabilities $P(D \in (a,b) \cap T \in (c,d))$ and the associated joint hazard function h(d,t). However, joint hazard functions can only be handled under rather restrictive assumptions (Heckman and Honoré 1989).

each time period, t = 0 …14. The baseline hazard increases with increasing distance. But, this monotonic behavior is disturbed by the time covariate: the positive[15] parameter suggests that, for points generated late in the process, the estimated hazard exceeds the baseline hazard. For points generated late in the process, the estimated hazard is smaller than the baseline hazard.

Figure 8: Estimated Hazard Function of d_{it}

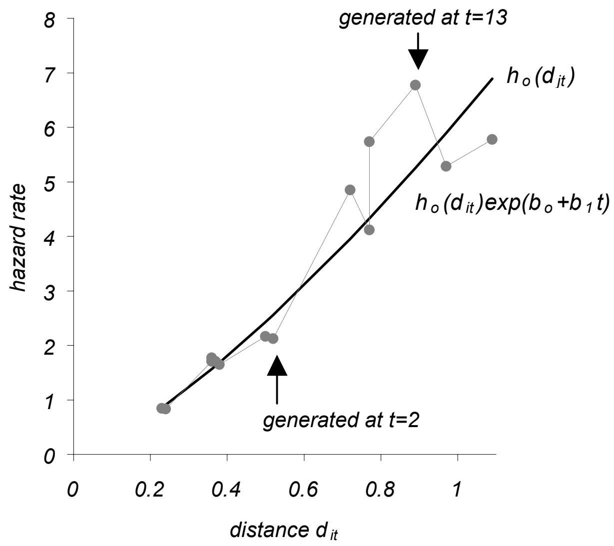


Table 2: Parameter Estimates (Standard Errors in Parentheses) of Distance Hazards for a Point Generation Trajectory

	Constant Speed	Acceleration	Slowing-down
Constant β_o	.299 (.219)	.189 (.303)	.363 (.184)
Time Parameter β_1	.015 (.026)	.068 (.076)	.004 (.015)
Scale: λ	1.514 (.189)	1.523 (.186)	1.511 (.194)
Shape: β	2.402 (.993)	2.457 (.987)	2.365 (.959)

Table 2 provides the estimates for three scenarios that differ with respect to the speed of the process but share the same trajectory as shown in Figure 7. Keeping the trajectory of point generation constant but altering the speed of the process will lead to changes in the magnitudes of the deviations from the baseline hazard, but not affect the baseline hazard or affect the overall shape of the estimated hazard function. Compared to the scenario of constant speed, acceleration increases the time parameter

[15] The parameter estimates reported in this paper refer to $h(d) = h_o(d)\ exp(\beta_o + \beta_1 d)$ rather than the more awkward specification $h(d) = h_o(d)\ exp(-\beta_o - \beta_1 d)$ used by Greene (1998).

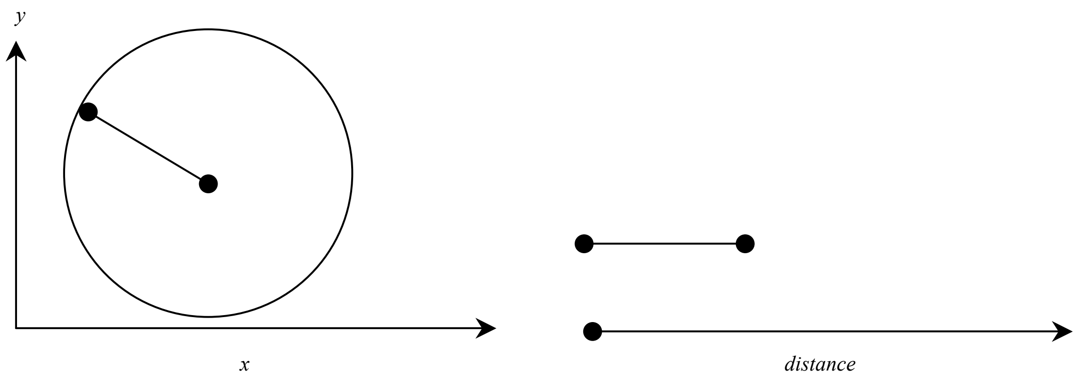
and thus increases the deviations from the baseline hazard, whereas a slowing-down of the process leads to a smaller effect of time on the distance hazard and thus decreases the deviations from the baseline hazard. It should be noted that changing the trajectory rather than the speed of the process possibly leads to a new set of observed nearest neighbor distance, d_{it}, and thus will affect the baseline hazard.

3.3 Movement in 'Medium Time' versus 'Medium Space'

The longitudinal framework focuses on modeling the conditional probability or hazard rate of an event happing in [t, t+δ], given that it has not yet happened at time t. Since every observation inevitably moves through the medium time, these conditional probabilities and hazards have a "natural" interpretation. For example, a hazard model simply shifts the focus from the probability of a person finding employment at time t, to the conditional probability of finding employment at time [t, t+δ] given that the job search was unsuccessful up to time t.

When hazard models are applied to distances between points rather than the duration of a spell, transferring the concept of 'passing through time' to a concept of 'moving through space' is not straightforward. Consequently, going beyond the description of a distance distribution and assigning explanatory value to the estimated models is rather problematic since the conceptual meaning of conditional probabilities and hazards becomes questionable.

Figure 9: Collapsing two-dimensional space into one-dimensional distance space



Two issues are particularly troublesome. The first deals with the identifiability of the 'ending point'. Space is measured via distance. Distance, however, is a projection of two-dimensional space to one-dimensional space and thus distance hazards actually refer to an infinite (and uncountable) set of points arranged in a circle around the observation, rather than just a specific point in space (Figure 9).

In order to deal effectively with this ambiguity of the 'medium space', one needs to uniquely identify the ending point on the circle with radius d (e.g., via the angle from a fixed reference point). Odland and Ellis (1992) take this approach even further. Very elegantly they utilize one of the advantages of spatial duration models, i.e., the models' ability to incorporate spatially varying covariates. Specifically, they use all spatial coordinates along the shortest line separating two nearest neighbor settlements and incorporate the coordinates as spatially varying covariates (i.e., as a function of distance) in a proportional hazard model. The disadvantage of this more sophisticated approach is, of course, that the parameters of spatially varying covariates are not easily interpreted beyond sign and significance.

Second, the transfer from the 'medium time' to the 'medium space' is also problematic because observations (points, settlements, firms, households) do not necessarily 'move' or 'search' through space. Even if one can conceptualize distances as a realization of some movement or search in space, the movements may be non-continuous and multi-directional. When analyzing spatial linkages (e.g., trade distances, moving distances) between a randomly selected point (e.g., firm, household) and a related point (client, new residence) the transfer from 'passing through time' to 'moving in space' requires assumptions about the nature of the underlying selection of a point (new client, new residence).

For example, suppose that a spatial hazard model is used to analyze distances between households' old and new residences. Assigning a meaningful interpretation to the conditional probability of the new residence being located at [9km, 9+δkm] given that it is at least 9 km away, presupposes that the residential search proceeds from the old residence in concentric circles of monotonically increasing radii. Such an interpretation is, however, not consistent with models of residential search behavior (e.g., Huff 1987).[16]

The concept of conditional probabilities or hazard rates for distances within spatial point patterns is even more problematic, if not impossible. A point pattern can be interpreted as one outcome of a point-generating process. Once the pattern is generated, nearest neighbor distances are "fixed" within the spatial structure. That is, once a point (be it part of the point pattern or randomly selected from anywhere in the study area[17]) is selected, the nearest neighbor distance is "predetermined" within the (exogenously given) spatial pattern, and the conditional probability of finding the nearest neighbor at distance d is either one or zero. In a temporal setting, this is of course not the case. For example, a randomly selected individual is not confronted with an exogenously given exit time but the exit can happen at any time. Thus, conceptually it makes sense to estimate the conditional probabilities in a temporal setting, whereas assigning a meaningful interpretation to hazards rates for nearest neighbor distances in a spatial point pattern is pointless.

Indicative for these conceptual differences is the kind of data that are being used for temporal versus spatial hazard models. For the temporal setting, the data are (ideally) panel data where randomly selected individuals are observed over time and their status (e.g., unemployed, not unemployed) is recorded through time. A spatial equivalent of this sort of data collection does not exist for the types of applications presented in the literature. There are, however, situations of spatial processes where data collection in a 'spatial panel' is appropriate and for which, consequently, the concept of a distance hazard has a natural interpretation.

These situations refer to processes characterized by a continuous expansion from a fixed source (the spatial analogue of the beginning of the "at-risk-period"; see Figure 1). At distance D=0, a state s_1 is observed, and at some distance away, a switch occurs to a state s_2 (the spatial analogue of the terminating event). The distance D=d at which the switch occurs (i.e., the terminating event) becomes the endogenous variable of interest.

[16] Residential search is an example where a hazard model can be meaningfully applied to the non-negative variable area, rather than distances.

[17] This distinction refers to the difference between nearest neighbor distance and point-individual distance (Cliff and Ord 1981).

For example, for the spread of a forest fire it is meaningful to determine the probability that, along a linear transsect, the fire will cease at distance [d, d+ δ] from its origin, given that it already approached to distance d. Urbanization and urban sprawl is another example of a continuous spread in space, and the data collection can indeed be thought of as a 'spatial panel' data collection. That is, for a randomly selected city and a randomly selected transsect originating at the city center, two states are recorded along the transsect: s_1 = urban land use, and s_2 = non-urban land use. Analyzing the distance at which the switch from s_1 to s_2 occurs is not only mathematically but also conceptually equivalent to analyzing the time at which a switch from one state to another occurs. Thus, the conditional probability or hazard of urban land use ending at distance [d, d+ δ] given that it persists at least until distance d has a meaningful interpretation. Since the concept of conditional probabilities and hazards has a meaningful interpretation in such situations, it will also be meaningful to estimate proportional hazard models that focus on factors that increase or decrease a baseline hazard.

The continuous spatial expansion –characteristic of the fire and urban sprawl examples– is not compatible with point patterns. The conceptual differences between the 'medium time' and the 'medium space' thus confine the use of spatial longitudinal models for point patterns to description, rather extend towards than explanation.

4. Summary and Conclusions

Over the last decade, the transfer of longitudinal models to spatial settings has entered the literature in the form of spatial hazard models. At the core of this transfer is a switch from 'duration' to 'distance' as the endogenous variable. The mathematical equivalence of duration and distance –both are non-negative continuous variables– has been used as justification to apply the rich methodological tool kit of longitudinal models to the analysis of distances in space.

This paper critically evaluates the literature on spatial hazard models and augments this literature by outlining the limitations as well as the as yet unexplored potential of longitudinal modeling in a spatial setting. Four main conclusions can be drawn. First, all existing applications of spatial duration models refer to distances measured between points, either in a point pattern (e.g., settlement pattern) or between points representing a spatial linkage (e.g., locations of firm and client). Other non-negative continuous variables that can describe space, such as area, have not been analyzed in a spatial hazard model.

Second, spatial hazard models can be nicely utilized to compare and to fit theoretical distributions to empirically observed distances. In the case of distances in a point pattern, spatial duration models also offer the advantage of dealing with edge effects via censoring. In the case of nearest neighbor distances, spatial duration models can be used to distinguish random from non-random (clustered or uniform) patterns by testing for a constant hazard of the squared nearest neighbor distances.

Third, since spatial hazard models allow us to control for exogenous variables, they can in particular account for the temporal component in the process that generates a spatial point pattern. The literature has not yet taken advantage of this opportunity and instead described distance hazards and survivor rates for completed point patterns only.

Fourth, the literature has also gone beyond describing distances in a spatial hazard framework and estimated explanatory models in the form of proportional hazard models for distances between points in a point pattern. Doing so presupposes that there is a conceptual meaning of spatial conditional probabilities and spatial hazards. This paper argues that a meaningful interpretation of hazards is problematic if the distances represent spatial linkages, and impossible if the distances refer to points in a point pattern. Instead, it is argued that a spatial hazard can only be meaningfully interpreted for spatial phenomena that spread continuously through space, i.e., phenomena for which a switch from one state to another can be observed as one proceeds away from the spatial origin. It is hoped that future research will provide empirical examples in this unexplored realm and thus make spatial hazard models part of the methodological tool kit of spatial scientists.

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